

09:20:11

OCA PAD AMENDMENT - PROJECT HEADER INFORMATION

08/01/91

Active

Project #: E-24-620	Cost share #: E-24-324	Rev #: 1
Center # : 10/24-6-R7198-0A0	Center shr #: 10/22-1-F7198-0A0	OCA file #:
		Work type : RES
Contract#: DMS-9100721	Mod #: BUD CH 6/24/91	Document : GRANT
Prime #:		Contract entity: GTRC
Subprojects ? : N		CFDA: 47.049
Main project #:		PE #: N/A

Project unit:	ISYE	Unit code: 02.010.124
Project director(s):		
BARNES E R	ISYE	(404)894-2310
AL-KHAYYAL F A	ISYE	(404)-

Sponsor/division names: NATL SCIENCE FOUNDATION / GENERAL
Sponsor/division codes: 107 / 000

Award period: 910601 to 921130 (performance) 930228 (reports)

Sponsor amount	New this change	Total to date
Contract value	0.00	39,919.00
Funded	0.00	39,919.00
Cost sharing amount		1,996.00

Does subcontracting plan apply?: N

Title: UNDERGRADUATE RESEARCH IN NUMERICAL LINEAR ALGEBRA AND OPTIMIZATION

PROJECT ADMINISTRATION DATA

OCA contact: Mildred S. Heyser 894-4820

Sponsor technical contact	Sponsor issuing office
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NATIONAL SCIENCE FOUNDATION
1800 G STREET, N.W.
WASHINGTON, D.C. 20550

Security class (U,C,S,TS) : U ONR resident rep. is ACO (Y/N): N
Defense priority rating : N/A NSF supplemental sheet
Equipment title vests with: Sponsor GIT X

Administrative comments -

TO REVISE BUDGET PER BUDGET CHANGE REQUEST DATED 6/24/91.(OPAS FORM)



GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 08/05/93

Project No. E-24-620

Center No. 10/24-6-R7198-0A0

Project Director BARNES E R

School/Lab ISYE

Sponsor NATL SCIENCE FOUNDATION/GENERAL

Contract/Grant No. DMS-9100721 Contract Entity GTRC

Prime Contract No.

Title UNDERGRADUATE RESEARCH IN NUMERICAL LINEAR ALGEBRA AND OPTIMIZATION

Effective Completion Date 921130 (Performance) 930228 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	N	
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

CommentsEFFECTIVE DATE 6-1-91. CONTRACT VALUE \$39,919.

Subproject Under Main Project No.

Continues Project No.

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Management	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other CARL BAXTER-FMD	Y
FRED CAIN-00D	Y

NATIONAL SCIENCE FOUNDATION
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WASHINGTON, DC 20550

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POSTAGE & FEES PAID
National Science Foundation
Permit No. G-89

PI/PO Name and Address

Ear. R. Barnes and Faiz Al-Khayyal
 School of Industrial and Systems Engineering
 Georgia Institute of Technology
 Atlanta, Georgia 30332-0205

NATIONAL SCIENCE FOUNDATION FINAL PROJECT REPORT

PART I - PROJECT IDENTIFICATION INFORMATION

1. Program Official/Org. Division of Mathematical Sciences

2. Program Name Research Experience for Undergraduates

3. Award Dates (MM/YY) **From:** 6/92 **To:** 8/94

4. Institution and Address

School of Industrial and Systems Engineering
 Georgia Institute of Technology
 Atlanta, GA 30332-0205

5. Award Number DMS - 9100721

6. Project Title

Linear Algebra and Optimization

This Packet Contains
NSF Form 98A
And 1 Return Envelope

PART IV – FINAL PROJECT REPORT – SUMMARY DATA ON PROJECT PERSONNEL

(To be submitted to cognizant Program Officer upon completion of project)

The data requested below are important for the development of a statistical profile on the personnel supported by Federal grants. The information on this part is solicited in response to Public Law 99-383 and 42 USC 1885C. All information provided will be treated as confidential and will be safeguarded in accordance with the provisions of the Privacy Act of 1974. You should submit a single copy of this part with each final project report. However, submission of the requested information is not mandatory and is not a precondition of future award(s). Check the "Decline to Provide Information" box below if you do not wish to provide the information.

Please enter the numbers of individuals supported under this grant.
Do not enter information for individuals working less than 40 hours in any calendar year.

	Senior Staff		Post-Doctorals		Graduate Students		Under-Graduates		Other Participants ¹	
	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.
A. Total, U.S. Citizens					1*		2	2		
B. Total, Permanent Residents							2			
U.S. Citizens or Permanent Residents ² :										
American Indian or Alaskan Native										
Asian.							1			
Black, Not of Hispanic Origin.										
Hispanic										
Pacific Islander										
White, Not of Hispanic Origin					1		3	2		
C. Total, Other Non-U.S. Citizens										
Specify Country										
1.										
2.										
3.										
D. Total, All participants (A + B + C)					1		4	2		
Disabled³										

☐ Decline to Provide Information: Check box if you do not wish to provide this information (you are still required to return this page along with Parts I-III).

¹Category includes, for example, college and precollege teachers, conference and workshop participants.

²Use the category that best describes the ethnic/racial status for all U.S. Citizens and Non-citizens with Permanent Residency. (If more than one category applies, use the one category that most closely reflects the person's recognition in the community.)

³A person having a physical or mental impairment that substantially limits one or more major life activities; who has a record of such impairment; or who is regarded as having such impairment. (Disabled individuals also should be counted under the appropriate ethnic/racial group unless they are classified as "Other Non-U.S. Citizens.")

AMERICAN INDIAN OR ALASKAN NATIVE: A person having origins in any of the original peoples of North America and who maintains cultural identification through tribal affiliation or community recognition.

ASIAN: A person having origins in any of the original peoples of East Asia, Southeast Asia or the Indian subcontinent. This area includes, for example, China, India, Indonesia, Japan, Korea and Vietnam.

BLACK, NOT OF HISPANIC ORIGIN: A person having origins in any of the black racial groups of Africa.

HISPANIC: A person of Mexican, Puerto Rican, Cuban, Central or South American or other Spanish culture or origin, regardless of race.

PACIFIC ISLANDER: A person having origins in any of the original peoples of Hawaii; the U.S. Pacific territories of Guam, American Samoa, and the Northern Marianas; the U.S. Trust Territory of Palau; the islands of Micronesia and Melanesia; or the Philippines.

WHITE, NOT OF HISPANIC ORIGIN: A person having origins in any of the original peoples of Europe, North Africa, or the Middle East.

*Teaching Assistant

Part II - Summary of Completed Project

We try to lure students into discovering the excitement of inventing and learning about new things. We try to select topics that well known mathematicians have written about and convince students that they can understand the material, and even add to what is known about the subject. We select topics in linear algebra that students learn about by their sophomore year. We also select topics for which we know new results can be obtained using elementary methods. We use computers extensively to solve many examples of problems using graphics whenever possible. In some cases patterns in the graphics point out phenomena we were not aware of and are not able to explain immediately. Students seem to get most excited about problems that arise in this way.

In each project we start students off with iterative techniques for solving equations. We convince them that the convergence of the scheme depends on properly choosing certain parameters in the equations. In the case of linear equations the parameters depend on the eigenvalues of the coefficient matrix. Our students have developed some techniques for estimating these eigenvalues in terms of the coefficients in the equations.

1 Overview

The focus of our work is to look at problems in mathematics where one is required to compute the largest or smallest value in a set of numbers that is defined implicitly. For example one might be required to find the shortest tour for a salesman who must visit n cities and return to his starting point. Or one might be required to find the largest eigenvalues of a real symmetric matrix. All of our problems are taken from optimization or from linear algebra. They have the common feature that while it may not be practical to enumerate all the numbers associated with one of our problems, it is a simple matter to compute certain averages of powers of these numbers. For example, if $A = (a_{ij})$ is a real symmetric matrix with eigenvalues $\lambda_1, \dots, \lambda_n$, then for any diagonal term a_{ii} , there exist nonnegative numbers p_1, \dots, p_n satisfying $p_1 + \dots + p_n = 1$ such that

$$a_{ii} = p_1 \lambda_1 + \dots + p_n \lambda_n.$$

We also have

$$\sum_{k=1}^n a_{ik}^2 = p_1 \lambda_1^2 + \dots + p_n \lambda_n^2.$$

In general, the average of the numbers $\lambda_1^r, \dots, \lambda_n^r$ with respect to the weights p_1, \dots, p_n is the number in position (i, i) of the matrix A^r . These averages are called moments of the eigenvalues.

There are also simple formulas for the moments of the values of the feasible solutions of certain optimization problems. In particular, there are formulas for certain moments of the possible values of the general quadratic assignment problem which includes the traveling salesman problem, certain plant location problems, and some graph partitioning and wiring problems that arise in laying out circuits on computer chips.

So our central question is: Given a few moments of a set of numbers, what can be said about the largest and smallest numbers in the set. Some results can be obtained by using the classical inequalities that students encounter in elementary courses in analysis. Some of these inequalities have geometric interpretations, especially for the eigenvalue problem, and students sometimes discover these during computer experiments with graphics software.

In the summer of 1992, there were six students in our program. We studied the problems described above. Our interest in the eigenvalue problem stems from our interest in solving linear systems of equations of the form $Ax = b$ by iterative schemes of the form

$$x^{k+1} = N^{-1}Px^k + N^{-1}b,$$

where $A = N - P$ is a symmetric splitting of A . When this scheme converges we can accelerate it if we know the extreme eigenvalues of $N^{-1}P$; or good estimates of them.

We were able to motivate all but one of our students to get involved with our project. In general, the students I am encountering in this project are of two types. Some of them are really good at mathematics and can discover things. Others are good with computers and can use them to shed light on many of the things we talk about. Both of these groups are fun to work with and contribute a lot to our project. The one student that we could not get to participate in our work belongs to a third class. In the three years we have run this program this is the only case where we have not been able to get full participation from our students. So I think this is a rare situation that does not warrant any special attention.

Our approach is to show students important results in linear algebra and optimization that have been obtained using elementary techniques, and to convince them that they can discover similar things. At this point in their careers we try to encourage them to do something interesting as apposed to something useful. For example, it would be very hard for us to compete with some of the software currently available for computing eigenvalues and for solving optimization problems. However, we can find interesting relationships between the eigenvalues of a matrix and the matrix entries. For example if $A = (a_{ij})$ is real symmetric, and if $\alpha < a_{ii} < \beta$ are numbers satisfying $(\beta - a_{ii})(a_{ii} - \alpha) \geq \sum_{j \neq i} a_{ij}^2$ there is an eigenvalue of A in the interval $[\alpha, \beta]$.

This summer we have a student working on the traveling salesman problem. We assume we have solved the problem for a small number of cities, say 5, and we then ask; over what region in the plane can we move one of the cities and have the current order for visiting the cities remain optimal? We don't know of any immediate application for this but it is an intriguing question. And students can learn something about the region by experimenting with simple computer graphics. In some cases we can obtain analytic expressions for the boundaries of the region. So our approach is to select problems from an area of mathematics where there are interesting applications and try to exhibit questions that are interesting and within the reach of undergraduates.

We believe we are having some impact. One of our students, Matthew Rudd, continued to work on his summer project as a senior thesis at Wake Forest this past year, and I have continued to consult with him on it. He is working on the problem of estimating the extreme eigenvalues of a matrix using information from smaller submatrices. I am especially pleased when students carry our projects back to their home institutions. Kimberly Weems from our 1991 summer did a senior thesis on eigenvalues at Spelman College. She is now in the Ph.D. program in mathematics at the University of Maryland. Yuriy Grinfeld also continued to work on our project back at Rutgers. He is now in the Ph.D. program in Operations Research there. We seem to make a difference when we are able to sufficiently motivate students so that they maintain an interest in our problems even after they leave us and we stop paying them.

2 Technical Results

Matthew Rudd found some bounds for the extreme roots of polynomials with real roots of polynomials with real roots which are analogous to the result that the extreme eigenvalues of a principal submatrix are bordered by the extreme eigenvalues of the matrix in the case of real symmetric matrices.

Ewald Hueffmeier studied the Kantorovich ratio for a symmetric matrix A . If $\lambda_1 \geq \dots \geq \lambda_n > 0$ are the eigenvalues of A the Kantorovich ratio for A is

$$K = \frac{\lambda_1 - \lambda_n}{\lambda_1 + \lambda_n}.$$

It governs the rate of convergence for certain iterative schemes for solving $Ax = b$. In the previous summer we showed that

$$K^2 \geq 1 - \frac{a_{ii}^2}{\sum_{k=1}^n a_{ik}^2}, i = 1, \dots, n. \quad (2.1)$$

Ewald showed that $K^2 \geq \min_{c \geq 0} f(c)$ where

$$f(c) = \max \left\{ \frac{(c - a_{ii})^2 + \sum_{k \neq i} a_{ik}^2}{c^2}, \frac{(c - a_{jj})^2 + \sum_{k \neq j} a_{jk}^2}{c^2} \right\}$$

for any fixed indices i and j . This result reduces to (2.1) when $i = j$. But in general it is sharper.

Pavel Grinfeld discovered a power method for computing the 1st and 2nd eigenvalues of a positive definite matrix. It is different than the usual power method and involves some clever inequalities for L_p norms. Pavel is in our program again this summer and is making major contributions.

Maria Theoharidis used Rouché's theorem to determine bounds on the zeros of polynomials in terms of their coefficients. For a long time this was a frustrating endeavor for her because she kept coming up with bounds that are known and obtainable by simpler methods. But back at school she was finally able to find a class of polynomials where her method gives better results than the well known ones. She wrote up a report about this and sent it to me.

Munho Yi provided computer support for the entire project. He compiled a huge amount of data analyzing the errors in our bounds. These gave us some sense of the power of our methods.